Lecture 4: Neutron Decay and the Physics of Beta Radiation: Notes

CBE 30235: Introduction to Nuclear Engineering — D. T. Leighton

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1 The Universal Law of Radioactive Decay (Lamarsh §2.9, 2.12)

Whether we are discussing the relaxation of an excited nucleus (Technetium), the decay of a free neutron, or the spontaneous splitting of Uranium, all radioactive processes follow the same fundamental statistical law.

1.1 The Governing Differential Equation

Radioactive decay is a **stochastic process**. We cannot predict when a specific nucleus will decay, but for a large population (N), the rate of decay is strictly proportional to the number of atoms present:

$$\frac{dN(t)}{dt} = -\lambda N(t) \tag{1}$$

Where:

- $N(t)$ is the number of radioactive nuclei at time t.
- λ is the **decay constant** (units of s^{-1}), representing the probability of decay per unit time.

1.2 The Exponential Decay Law

Integrating the governing equation from $t = 0$ to t yields the standard decay formula:

$$N(t) = N_0 e^{-\lambda t} \tag{2}$$

1.3 The Stochastic Assumption

The decay law $N(t) = N_0 e^{-\lambda t}$ relies on two fundamental assumptions:

1. **Independence:** The probability of a nucleus decaying is independent of its environment (temperature, pressure, chemical bonding) and independent of the behavior of neighboring nuclei.

2. **Memoryless:** A nucleus that has existed for 1,000 years has the exact same probability of decaying in the next second as a nucleus that was just created.

Note on Fission: While *spontaneous* fission follows this law, the *induced* fission we see in reactors is a collision-based reaction. In that case, the rate is governed by the intensity of the neutron beam rather than just a constant decay probability λ.

1.4 Half-Life ($T_{1/2}$) and Mean Life (τ)

While λ is mathematically convenient, we often characterize isotopes by their **half-life**: the time required for the population to decrease by half.

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \implies T_{1/2} = \frac{\ln(2)}{\lambda} \approx \frac{0.693}{\lambda} \tag{3}$$

The **mean life** (τ) is the reciprocal of the decay constant ($\tau = 1/\lambda$) and represents the average life expectancy of a single nucleus.

1.5 Activity (A)

In the lab, we rarely count atoms; instead, we measure the rate of radiation emission, known as **Activity**:

$$A(t) = \left|\frac{dN}{dt}\right| = \lambda N(t) = A_0 e^{-\lambda t} \tag{4}$$

Units of Activity:

- **Becquerel (Bq):** 1 decay/second (The SI unit).
- **Curie (Ci):** 3.7×10^{10} Bq (Originally defined as the activity of 1 gram of ^{226}Ra).

Engineering Context: In nuclear medicine, like the Technetium stress test, the "dosage" is prescribed in Activity (e.g., 20 mCi). Because the half-life is short (6 hours), the engineer must account for the decay that occurs between the time the dose is "milked" from the generator and the time it is injected into the patient.

2 Case Study: The Free Neutron

Having established the universal law $N(t) = N_0 e^{-\lambda t}$, we now apply it to the simplest "unstable" particle: the free neutron. While a neutron is stable when bound inside a nucleus, it begins its "stochastic clock" the moment it is liberated.

2.1 Energy Accounting: The Mass Threshold

The fundamental reason a free neutron decays is a matter of mass-energy accounting. Using the neutral atom convention:

- **Mass of Neutron (m_n):** 1.008665 u
- **Mass of Hydrogen-1 ($M(^1\mathbf{H})$):** 1.007825 u (Includes $m_p + m_e$)

The Q-value (energy released) is:

$$Q = [m_n - M(^1\text{H})] \times 931.5 \text{ MeV/u} = 0.782 \text{ MeV} \tag{5}$$

Because $Q > 0$, the transition is energetically favorable. The neutron spontaneously transforms via **Beta Decay**:

$$n \longrightarrow p + e^- + \bar{\nu}_e + 0.782 \text{ MeV} \tag{6}$$

2.2 The Stability Paradox

If the neutron is inherently unstable, why does the universe not consist entirely of protons?

- **The Potential Well:** Inside a nucleus, the neutron is trapped in a deep potential well created by the Strong Force.
- **The Energy Barrier:** For a bound neutron to decay, the resulting nucleus $(Z+1, N-1)$ must have a lower total mass than the original nucleus. In stable isotopes like ^{12}C, the "cost" of the new configuration exceeds the 0.782 MeV released, effectively "freezing" the neutron in a stable state.

2.3 Neutron Kinetics

Applying our Section 1 definitions to the free neutron:

- **Mean Life (τ):** Approximately 880 seconds.
- **Half-Life ($T_{1/2}$):**

$$T_{1/2} = \tau \ln(2) \approx 610 \text{ seconds } (\sim 10.2 \text{ minutes}) \tag{7}$$

Engineering Note: In a nuclear reactor, the life of a neutron is much shorter than 10 minutes. A neutron typically survives for only microseconds to milliseconds before it is absorbed by a nucleus or leaks out of the core. Thus, in reactor kinetics, we often treat the neutron as "immortal" regarding decay, focusing instead on its absorption probability.

Supplemental Resources: The Physics of the Free Neutron

Because the instability of the free neutron is a fundamental precursor to understanding Beta decay, it is useful to read the Wikipedia article on free neutron decay. It has a nice table showing how understanding of the decay has changed over the years, including how the decay can be best described in terms of transmutation of a down quark. Interestingly, despite decades of study even the half-life of a neutron has not been fully resolved, with significantly different values obtained via different measurement techniques (e.g., the "Neutron Lifetime Puzzle").

3 Mechanics of Beta Decay (β^-) (Lamarsh §2.8)

When we move from an isolated neutron to a nucleus, the decay process is constrained by the nuclear environment, yet it remains the primary mechanism for nuclei to adjust their N/Z ratio toward the Valley of Stability.

3.1 The Isobaric Transition

In β^- decay, a neutron transforms into a proton. This results in a transition along a **line of isobars** (constant A), moving one step "down and to the right" on the Chart of the Nuclides:

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e + Q \tag{8}$$

Example: The decay of Carbon-14 (used in dating) into Nitrogen-14:

$$^{14}_{6}\text{C} \longrightarrow {}^{14}_{7}\text{N} + e^- + \bar{\nu}_e + 0.156 \text{ MeV} \tag{9}$$

3.2 The Neutrino and the Energy Spectrum

The discovery of the neutrino is a classic example of the conservation laws (Energy and Momentum) dictating the existence of unseen physics.

- **The Observation:** Unlike α particles, which are mono-energetic, electrons from a specific β decay source emerge with a *continuous* distribution of kinetic energies.
- **The Distribution:** The Q-value represents the maximum possible energy (E_{max}). The electron and the antineutrino share this energy stochastically.
- **Rule of Thumb:** For most engineering calculations, the average kinetic energy of the emitted electron is taken as:

$$E_{avg} \approx \frac{1}{3} E_{max} \tag{10}$$

3.3 Engineering Implications: Shielding and Dose

The continuous spectrum presents unique challenges in radiation protection:

1. **Range vs. Energy:** While β particles are easily stopped (a few meters in air, millimeters in plastic), they present a significant **shallow dose** hazard. They can penetrate the dead outer layer of skin to damage the living basal layer, resulting in "Beta burns" similar in appearance to a severe sunburn.
2. **Bremsstrahlung (Braking Radiation):** This is the "hidden" danger. When high-energy electrons are rapidly decelerated by a shield, they emit X-rays.
   - *The Rule:* To minimize X-ray production, β emitters should be shielded with **low-Z materials** (like Acrylic/Plexiglass or Aluminum) rather than Lead.
3. **Internal Dosimetry:** If a β emitter is ingested, the E_{avg} is used to calculate the total energy deposited in the organ, as the neutrino escapes the body entirely without interacting.

Supplemental Resources: The Beta Mystery

Lamarsh provides the "what," but these resources explain the "how" of the neutrino's discovery and the physics of the spectrum:

- **The American Physical Society (APS):** Pauli's Invention of the Neutrino – A historical account of the "desperate remedy" required to save the law of conservation of energy.
- **HyperPhysics (Georgia State University):** Beta Decay Concepts – A clear visualization of the energy distribution and the Q-value calculations.
- **Wikipedia:** The Beta Particle – Includes a useful discussion on range, shielding, and the Bremsstrahlung effect in engineering practice.

4 Positron Emission (β^+) and Electron Capture (EC)

If a nucleus has "too many" protons for stability (islands of proton-rich isotopes), it must convert a proton into a neutron to move toward the Valley of Stability.

4.1 Positron Emission (β^+)

A proton in the nucleus transforms into a neutron, emitting a positron (e^+) and a neutrino (ν_e):

$$^A_Z X \longrightarrow {}^A_{Z-1}Y + e^+ + \nu_e + Q \tag{11}$$

The 1.022 MeV "Entry Fee": This is a critical calculation for nuclear engineers. Because the neutron is heavier than the proton, and we are creating a positron, the parent atom must be significantly heavier than the daughter.

- When using *atomic* masses, the Q-value formula is: $Q = [M(X) - M(Y) - 2m_e]c^2$.
- The parent must be at least **1.022 MeV** (the mass of two electrons) heavier than the daughter for β^+ to be energetically possible.

4.2 Electron Capture (EC)

What happens if the energy difference is less than 1.022 MeV? The nucleus "cheats" by capturing an orbital electron (usually from the K-shell) to convert the proton:

$$^A_Z X + e^- \longrightarrow {}^A_{Z-1}Y + \nu_e \tag{12}$$

- **Secondary Radiation:** EC leaves a hole in the electron shell. When an outer electron drops down to fill it, a **Characteristic X-ray** or an **Auger electron** is emitted. This is the only way we can "see" that EC has occurred.

4.3 Engineering Application: PET Imaging

Positron emission is the basis for **Positron Emission Tomography (PET)**.

1. A patient is injected with a β^+ emitter like Fluorine-18.
2. The emitted positron travels a few millimeters before it meets an electron.
3. **Annihilation:** The two particles vanish, and their mass is converted into two **511 keV gamma rays** emitted exactly 180° apart.
4. The scanner uses "coincidence detection" to draw a line between the two hits, pinpointing the source.

Supplemental Resources: PET Imaging and Annihilation Physics

Since the clinical application of positron emission is outside the scope of Lamarsh, the following resources provide the engineering and physical context for PET scans:

- **Wikipedia:** Positron Emission Tomography – An excellent technical overview of the "Coincidence Detection" engineering that allows scanners to ignore background radiation.
- **Mayo Clinic:** Positron emission tomography scan – Provides information for patients about the technique.

5 Exit Hook: The Carbon-14 "Nuclear Clock"

We have spent today looking at the mechanics of β^- decay. We saw how ${}^{14}_{6}\text{C}$ transforms into ${}^{14}_{7}\text{N}$ by emitting an electron and an antineutrino. But for a Chemical Engineer, the most interesting question isn't *how* it decays, but how it got there in the first place and why it stays in equilibrium.

5.1 The Breath of the Atmosphere

Carbon-14 is not "primordial" (left over from the earth's formation). It is constantly being created in the upper atmosphere when cosmic ray neutrons smash into Nitrogen-14:

$$ {}^{1}_{0}n + ^{14}_{7}\text{N} \longrightarrow {}^{14}_{6}\text{C} + ^{1}_{1}p \tag{13} $$

This ${}^{14}\text{C}$ quickly oxidizes into ${}^{14}\text{CO}_2$ and enters the food chain.

5.2 The Steady-State Assumption

Because ${}^{14}\text{C}$ is created at a roughly constant rate and decays with a half-life of **5,730 years**, the ratio of ${}^{14}\text{C}$ to stable ${}^{12}\text{C}$ in the atmosphere (and in your body right now) is in a steady state.

5.3 The "Stopwatch" Starts at Death

The moment a plant or animal dies, it stops "breathing" new ${}^{14}\text{C}$. The existing "inventory" begins to decay away according to our law: $N(t) = N_0 e^{-\lambda t}$. By measuring the remaining activity, we can calculate exactly how long ago the organism stopped exchanging carbon with the atmosphere.

> *Preview for Friday:* We will dive into the math of the Carbon-14 clock. We will calculate the "Specific Activity" of a human being, discuss why the Industrial Revolution (and nuclear testing) "broke" the clock, and see how we can use Beta decay to date everything from the Dead Sea Scrolls to ancient wine.

Beta Decay Stability Analysis: Carbon-13 vs. Carbon-14

In β^- decay, a neutron is converted into a proton, an electron, and an anti-neutrino. The decay is energetically possible only if the atomic mass of the parent (Z) is greater than the atomic mass of the daughter ($Z+1$). We use the conversion factor $1\,\text{u} = 931.494\,\text{MeV}/c^2$.

1. Carbon-13 (^{13}C)

The hypothetical β^- decay of Carbon-13 would produce Nitrogen-13:

$$^{13}_{6}\text{C} \rightarrow {}^{13}_{7}\text{N} + e^- + \bar{\nu}_e$$

Mass Data:

- Atomic mass of ^{13}C: $13.003355\,\text{u}$
- Atomic mass of ^{13}N: $13.005739\,\text{u}$

Energy Calculation:

$$\begin{aligned} Q &= [M(^{13}\text{C}) - M(^{13}\text{N})]c^2 \\ Q &= [13.003355 - 13.005739]\,\text{u} \times 931.494\,\text{MeV/u} \\ Q &\approx -2.221\,\text{MeV} \end{aligned}$$

Conclusion: Because $Q < 0$, the decay is energetically forbidden. ^{13}C is a stable isotope.

2. Carbon-14 (^{14}C)

The β^- decay of Carbon-14 produces Nitrogen-14:

$$^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + e^- + \bar{\nu}_e$$

Mass Data:

- Atomic mass of ^{14}C: $14.003242\,\text{u}$
- Atomic mass of ^{14}N: $14.003074\,\text{u}$

Energy Calculation:

$$\begin{aligned} \Delta m &= M_{parent} - M_{daughter} \\ \Delta m &= 14.003242 - 14.003074 = 0.000168\,\text{u} \\ Q &= 0.000168\,\text{u} \times 931.494\,\text{MeV/u} \\ Q &\approx 0.156\,\text{MeV (or } 156\,\text{keV)} \end{aligned}$$

Conclusion: Because $Q > 0$, the decay is spontaneous. This $156\,\text{keV}$ is the maximum kinetic energy shared between the electron and the anti-neutrino.

3. Binding Energy per Nucleon (BE/A) Comparison

While the Q-value tells us if a decay *can* happen, the Binding Energy (BE) tells us how tightly the nucleons are held together. We calculate BE by comparing the mass of the atom to its individual constituents: 6 or 7 Hydrogen atoms ($m_H = 1.007825\,\mathrm{u}$) and the remaining neutrons ($m_n = 1.008665\,\mathrm{u}$).

Carbon-13 vs. Nitrogen-13

- **^{13}C ($Z = 6, N = 7$):**

$$BE = [6(1.007825) + 7(1.008665) - 13.003355] \times 931.494 = 97.108\,\mathrm{MeV}$$

$$BE/A = 97.108/13 \approx \mathbf{7.470}\,\mathrm{MeV/nucleon}$$

- **^{13}N ($Z = 7, N = 6$):**

$$BE = [7(1.007825) + 6(1.008665) - 13.005739] \times 931.494 = 94.105\,\mathrm{MeV}$$

$$BE/A = 94.105/13 \approx \mathbf{7.239}\,\mathrm{MeV/nucleon}$$

Result: ^{13}C is more tightly bound. This confirms why it is the stable "sink" for this mass number.

Carbon-14 vs. Nitrogen-14

- **^{14}C ($Z = 6, N = 8$):**

$$BE = [6(1.007825) + 8(1.008665) - 14.003242] \times 931.494 = 105.285\,\mathrm{MeV}$$

$$BE/A = 105.285/14 \approx \mathbf{7.520}\,\mathrm{MeV/nucleon}$$

- **^{14}N ($Z = 7, N = 7$):**

$$BE = [7(1.007825) + 7(1.008665) - 14.003074] \times 931.494 = 104.659\,\mathrm{MeV}$$

$$BE/A = 104.659/14 \approx \mathbf{7.476}\,\mathrm{MeV/nucleon}$$

4. Positron Decay (β^+) Analysis: Nitrogen-13

Nitrogen-13 is a proton-rich isotope that decays into stable Carbon-13 via positron emission. For β^+ decay to be spontaneous, the mass difference must exceed two electron masses ($2m_e \approx 1.022\,\mathrm{MeV}$).

$$^{13}_{7}\mathrm{N} \to {}^{13}_{6}\mathrm{C} + e^+ + \nu_e$$

Mass Data:

- Atomic mass of ^{13}N: 13.005739 u
- Atomic mass of ^{13}C: 13.003355 u
- Electron mass (m_e): 0.0005486 u

Energy Calculation (Q-value):

$$\begin{aligned}
Q &= [M(^{13}\text{N}) - M(^{13}\text{C}) - 2m_e]c^2 \\
\Delta M_{atomic} &= 13.005739 - 13.003355 = 0.002384\,\text{u} \\
Q &= (0.002384 - 0.001097)\,\text{u} \times 931.494\,\text{MeV/u} \\
Q &\approx \mathbf{1.198}\,\text{MeV}
\end{aligned}$$

Conclusion: Since $Q > 0$, the decay is spontaneous. The positron carries away kinetic energy, eventually annihilating with an electron to produce two 511 keV gamma rays.

Uses of N13

Nitrogen-13 has a very short half-life of 9.965 minutes. Because of this short lifespan, N13 is famous in modern medicine as a tracer in PET (Positron Emission Tomography) scans. It is usually produced on-site at hospitals using a cyclotron to track blood flow through the heart.

Nuclear Analysis: Fluorine-18 (^{18}F)

Fluorine-18 is the workhorse of Positron Emission Tomography (PET). It is a synthetic radioisotope with a half-life of $t_{1/2} = 109.77$ minutes.

1. Decay Analysis (β^+ Emission)

The primary decay mode of ^{18}F is positron emission to stable ^{18}O.

$$^{18}_{9}\mathrm{F} \rightarrow {}^{18}_{8}\mathrm{O} + e^+ + \nu_e$$

Mass Data:

- Atomic mass of ^{18}F: 18.000938 u
- Atomic mass of ^{18}O: 17.999161 u
- Electron mass (m_e): 0.0005486 u

Energy Calculation (Q-value):

$$\begin{aligned}
Q_{\text{decay}} &= [M(^{18}\mathrm{F}) - M(^{18}\mathrm{O}) - 2m_e]c^2 \\
\Delta M &= (18.000938 - 17.999161 - 0.001097)\,\mathrm{u} \\
Q_{\text{decay}} &= 0.000680\,\mathrm{u} \times 931.494\,\mathrm{MeV/u} \approx \mathbf{0.633}\,\mathrm{MeV}
\end{aligned}$$

Note: The $2m_e$ penalty accounts for the lost positron and the lost orbital electron required to reach a neutral ^{18}O state.

2. Cyclotron Production: $^{18}\mathbf{O}(p, n)^{18}\mathbf{F}$

To produce ^{18}F, enriched "heavy water" ($H_2{}^{18}O$) is bombarded with protons.
Reaction Energy Balance (Q_{rxn}):

$$\begin{aligned}
Q_{rxn} &= [M(^{18}\mathrm{O}) + M(^{1}\mathrm{H}) - M(^{18}\mathrm{F}) - M(n)]c^2 \\
Q_{rxn} &= [17.999161 + 1.007825 - 18.000938 - 1.008665] \times 931.494 \\
Q_{rxn} &\approx -\mathbf{2.438}\,\mathrm{MeV}\ \text{(Endothermic)}
\end{aligned}$$

Threshold Energy (E_{th}): To conserve momentum in the lab frame, the incident proton must have a minimum kinetic energy:

$$E_{th} = |Q_{rxn}| \left(\frac{m_p + m_O}{m_O} \right) = 2.438 \left(\frac{19}{18} \right) \approx \mathbf{2.573}\,\mathrm{MeV}$$

Coulomb Barrier (V_c):

$$V_c \approx \frac{1.44 \cdot Z_p Z_O}{1.25(1^{1/3} + 18^{1/3})} \approx \mathbf{2.55}\,\mathrm{MeV}$$

Conclusion: The proton must overcome a total barrier of approximately $2.6\,MeV$. In clinical practice, 11–18 MeV cyclotrons are used to maximize the reaction cross-section.

Isotope	Half-life ($t_{1/2}$)	Common Reaction	Primary Clinical Use
Oxygen-15	2.04 min	$^{14}N(d, n)^{15}O$	Water/Cerebral Blood Flow
Nitrogen-13	9.97 min	$^{16}O(p, \alpha)^{13}N$	Ammonia/Myocardial Perfusion
Carbon-11	20.3 min	$^{14}N(p, \alpha)^{11}C$	Brain Receptor Mapping
Fluorine-18	109.8 min	$^{18}O(p, n)^{18}F$	Oncology (FDG Metabolism)

Table 1: Comparison of common positron-emitting radionuclides.

6. Comparative Summary of Clinical PET Isotopes

The following table summarizes the four primary positron emitters used in modern molecular imaging. Note the direct correlation between half-life and the logistical "reach" of the isotope.

Logistical Engineering Analysis

From a systems perspective, these isotopes represent different "Operational Envelopes":

- **On-Site Only (^{15}O, ^{13}N):** Must be produced in a cyclotron directly adjacent to the PET scanner. The production-to-injection window is measured in seconds.
- **Regional Hub (^{18}F):** Can be produced at a central "Isotope Pharmacy" and driven via courier to hospitals within a 2-3 hour radius (e.g., Chicago to South Bend).
- **Energy vs. Resolution:** Higher Q-values (like in ^{15}O) result in higher-energy positrons, which travel further in tissue before annihilation, technically limiting the spatial resolution compared to the "softer" ^{18}F decay.